On the integration of acoustic phase-gradient metasurfaces in aeronautics

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Metamaterials ...

Macroscopic composite of periodic or non-periodic structures, able to produce responses to a specific excitation not available in nature by conventionally engineered materials, that derive their properties from the designed structures and geometries more than from their chemical composition, and the behavior of properly defined unit cells can be translated into averaged effective parameters.

...and Metasurfaces

A metamaterial device built with a thickness (significantly) below the working wavelength is usually called metasurface. Still its behaviour can be translated into averaged effective parameters.
What is a Metabebehaviour?

- negative bulk modulus
- acoustic black hole
- extraordinary absorption
- extraordinary reflection
- acoustic invisibility
- negative mass
- double negativity
- negative refractive index
- zero index
- extraordinary refraction

...and other exotic properties still to be imagined...
Generalized Snell’s Law

\[
\sin \theta_r = \frac{\lambda}{2\pi} \frac{\partial}{\partial x} \Delta \phi(x, \lambda) + \sin \theta_i
\]

\[
n_2 \sin \theta_t = \frac{\lambda}{2\pi} \frac{\partial}{\partial x} \Delta \phi(x, \lambda) + n_1 \sin \theta_i
\]
Generalized Snell’s Law

\[ \sin \theta_r = \frac{\lambda}{2\pi} \frac{\partial}{\partial x} \Delta \phi(x, \lambda) + \sin \theta_i \]
Standard Transformation Acoustics

Mapping: \( \Omega \to \omega \quad x = \chi(X) \)

Deformation gradient: \( F = \nabla x \quad \Lambda = \det F \)

Polar decomposition: \( F = VR \quad V^2 = FF^T \)

Laplacian: \( \nabla^2 f = \Lambda \text{div} \Lambda^{-1} V^2 \nabla f \)

Effective parameters:
\[
\begin{align*}
\frac{\partial^2 p}{\partial t^2} - \frac{\kappa_0}{\rho_0} \nabla^2 p &= 0, \quad x \in \Omega \\
\frac{\partial^2 p}{\partial t^2} - \kappa \text{div} \ \rho^{-1} \nabla p &= 0, \quad x \in \omega
\end{align*}
\]
\[
\kappa = \kappa_0 \Lambda, \quad \rho = \rho_0 \Lambda \nu^{-2}
\]
This work

- Enlighten the **link** between STA and GSL
- Effective parameters of a **metafluid** for extraordinary reflection
- Optimized metasurface (with and w/o flow) for **field shaping**
- Towards (possible) **integration** with standard liners and metamaterials

Long term goal

- **Virtual scarfin**g for aeronautical nacelles: reducing ground-directed directivity
- **Performance enhancement** of existing acoustic treatments
GSL-based Metasurface: basic example

\[
\sin \theta_r = \frac{\lambda}{2\pi} \frac{\partial}{\partial x} \Delta \phi(x, \lambda) + \sin \theta_i
\]

Easiest implementation \rightarrow straight wave-guides

\[
\Delta \phi = 2 \frac{2\pi}{\lambda} h
\]
Transformation Acoustics: GSL-based Metasurface (cont’d)

\[
\begin{align*}
x' &= x \\
y' &= \frac{a(x)}{b(x)} y + q(x)
\end{align*}
\]

\[
\begin{bmatrix}
F_{11} &= \frac{a(x)}{b(x)} \\
F_{12} &= 0 \\
F_{21} &= 0 \\
F_{22} &= 1
\end{bmatrix}
\]

Effective parameters

\[
\kappa = \frac{a(x)}{b(x)} \kappa_0, \quad \rho = \frac{b(x)}{a(x)} \rho_0
\]
Transformation Acoustics: GSL-based Metasurface (cont’d)

Wave guides  |  Discrete metafluid  |  Continuous metafluid
Numerical Setup

**Finite Element Method Analysis**

Acoustic propagation in a **uniform** moving medium

Partially **metasurface-lined** duct ($f^* = 3430$ Hz)

Incident field from a **monopolar point source** inside the duct

\[ \begin{align*}
- \frac{\partial}{\partial t} \left[ \frac{\rho_0}{c_0^2} \left( \frac{\partial \phi}{\partial t} + \mathbf{v}_0 \cdot \nabla \phi \right) \right] + \nabla \cdot \left[ \rho_0 \nabla \phi - \frac{\rho_0}{c_0^2} \left( \frac{\partial \phi}{\partial t} + \mathbf{v}_0 \cdot \nabla \phi \right) \mathbf{v}_0 \right] &= 0
\end{align*} \]

\[ K_i, Q_i \text{ optimized: } \max \left( \int_{S^*} \mathbf{IL} dS \right) \]
Numerical Setup

**Finite Element Method Analysis**

Acoustic propagation in a **uniform** moving medium

Partially **metasurface-lined** duct ($f^* = 3430$ Hz)

Incident field from a **monopolar point source** inside the duct

\[
\begin{align*}
-\frac{\partial}{\partial t} \left[ \rho_0 c_0^2 \left( \frac{\partial \varphi}{\partial t} + v_0 \cdot \nabla \varphi \right) \right] + \nabla \cdot \left[ \rho_0 \nabla \varphi - \frac{\rho_0}{c_0^2} \left( \frac{\partial \varphi}{\partial t} + v_0 \cdot \nabla \varphi \right) \right] v_0 = 0
\end{align*}
\]

\[K_i, Q_i \text{ optimized: } \max \left( \int_{S^*} IL dS \right)\]
Numerical Optimization

Deterministic Particle Swarm Optimizer

Based on the social–behavior metaphor of a flock of birds searching for food

Deterministic

Derivative Free

\[ \mathbf{v}_i^{k+1} = \chi [\mathbf{v}_i^k + c_1 (\mathbf{p}_i - \mathbf{x}_i^k) + c_2 (\mathbf{g} - \mathbf{x}_i^k)] \]

\[ \mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \]

\[ \mathbf{x}_i = [x_1, x_2, \ldots, x_{N_c}]^T \]
Results: pressure fields

M=0.0

Best Function Value: 0.00115729

M=0.3

Best Function Value: 0.00151967

M=0.0

M=0.3

Opt MS @ M=0.0

Opt MS@ M=0.0

Opt MS @ M=0.3
Results: IL HW vs. MS

M=0.0

M=0.3
Results: IL HW vs. MS

\[ IL = 10 \log_{10} \left( \frac{p_{HW}^2}{p_{MS}^2} \right) \]
Results: IL HW vs. MS

\[ IL = 10 \log_{10} \left( \frac{p_{HW}^2}{p_{MS}^2} \right) \]
An exercise: integration with perfect absorbers

\[ z_{\text{absorbers}} = 1.0 + i0.0 \]

PG metasurfaces
Results: IL HW vs. PA+MS

M=0.0

M=0.3
Results: IL HW vs. PA+MS

\[ IL = 10 \log_{10} \left( \frac{p_{HW}^2}{p_{PA+MS}^2} \right) \]
Results: IL HW vs. PA+MS

\[ IL = 10 \log_{10} \left( \frac{p_{HW}^2}{p_{PA+MS}^2} \right) \]
Results: IL on S*

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<tr>
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<th>M=0.0</th>
<th>M=0.3</th>
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<tbody>
<tr>
<td>MS vs. PA+MS</td>
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<tr>
<td>PA vs. PA+MS</td>
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<tr>
<td>PA v2 vs. PA+MS</td>
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\[
\frac{\int_S p_{\text{LINE}} dS}{\int_S p_{\text{HWD}} dS}
\]

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<td>33%</td>
<td>43%</td>
<td>68%</td>
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<tr>
<td>M=0.3</td>
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<td>49%</td>
<td>64%</td>
<td>72%</td>
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Horizon 2020

AdvancEd aircraft-noise-Alleviation devIceS using meTamaterials

23rd CEAS-ASC Workshop 2019, Roma 26/09/2019
An exercise: integration with perfect absorbers

Same lining length: a fair comparison!

PA* (PA*: same length of PA+MS)

PA+MS** (MS**: optimized MS in the presence of PA)
Results: IL PA* vs. PA+MS**

IL = 10 \log_{10} \left( \frac{p_{PA}^2}{p_{PA+MS}^2} \right)
Results: IL PA* vs. PA+MS**

\[ IL = 10 \log_{10} \left( \frac{p_{PA}^2}{p_{PA+MS}^2} \right) \]
Remarks and future development

**Achievement**

- Equivalent metafluids for GSL-based metasurfaces
- Framework for efficient phase profile optimization (with flow)
- Preliminary integration of optimized MS in a 2D duct

**On going activities**

- More realistic geometry
- More realistic sources
- More realistic flow
- Validation against experiments in presence of flow
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