Slow sound laser in lined flow ducts

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Analogies

Waves in a spacetime

$$\partial_\mu \sqrt{|g|} g^\mu_\nu \partial_\nu \phi = 0$$

Picture of a Black Hole
(From: Event Horizon Telescope ’19)

Acoustic waves with mean flow

$$\left( \partial_t + \nabla \cdot \mathbf{v} \right) \frac{\rho}{c^2} \left( \partial_t + \mathbf{v} \cdot \nabla \right) \phi - \nabla \cdot \rho \nabla \phi = 0$$
What happens when flow becomes faster than waves?

- Wave trapping
  - acoustic analogue of a black hole: *Dumb hole*
- Exotic wave effects around black holes could be reproduced
What happens when flow becomes faster than waves?

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“Slow sound”: an opportunity to realize dumb holes?
Speed of sound:

\[ c_0 = \sqrt{\frac{K}{\rho_0}} \]

\( K \): isentropic bulk modulus
\( \rho_0 \): density

Slow sound:
- Tubes mounted flush → **Metamaterial like**
- **Lower the effective stiffness**
- Long wavelengths:
  \( c_{\text{eff}} < c_0 \)

Waveguide with impedance treatment
Wave propagation in lined ducts

- Units where duct height $H = 1$ and $c_0 = 1$
- Potential perturbations $\mathbf{v} = \nabla \phi$
- Inside the duct

$$\partial_t^2 \phi - \Delta \phi = 0.$$
Wave propagation in lined ducts

Boundaries:

- $y = 0$ hard wall: $\partial_y \phi = 0$
- $y = 1$ impedance wall: $p = Z \partial_y \phi$
- Complex impedance $Z$. Tube model:

$$Z(\omega) = \frac{i}{\sigma \tan(b \omega)} + \Gamma$$

- Dissipation $\Gamma$ neglected
- Low frequencies $\omega \ll \frac{\pi}{2b}$ (subresonance)
Fixed frequency $\omega = 2\pi f$

Two acoustic modes

$\omega \ll \pi/2b$:

$$\omega^2 = \frac{k^2}{1 + b}$$

Hence $c_{\text{eff}} = 1/\sqrt{1 + b}$

Increase $\omega \rightarrow$ dispersion
Lined ducts with mean flow

- Mean Mach number $M_0 = U_0/c_0$
- **Two** types of flow:
  - **Subsonic:** $M_0\sqrt{1 + b} < 1$
  - **Supersonic:** $M_0\sqrt{1 + b} > 1$
- Possible for $M_0 < 1$
- What is the dispersion relation?
Lined ducts with mean flow

\[ \begin{align*}
&\text{Inside the duct} \\
&(\partial_t + M \partial_x)^2 \phi - \Delta \phi = 0. \\
&\text{Hard problem: vorticity modes, inhomogeneities, etc.} \\
&M \approx M_0 \text{ constant except in boundary layer } 1 - \delta \lesssim y < 1
\end{align*} \]
Lined ducts with mean flow

**Asymptotic approach:** effective boundary condition

[Brambley ’13, AC, Aurégan, Pagneux ’19]

**Our boundary condition**

- pressure $p$ is continuous
- displacement $\eta$ is continuous
- **But** effective compliance

$$C_{\text{eff}}(\omega) = \frac{\eta}{p} = C_0(\omega) + \delta C_1(\omega)$$

- $\delta = 0$ is Ingard-Myers boundary conditions
Lined ducts with mean flow

**Subsonic flows $M_0 < c_{\text{eff}}$**

- Two acoustic modes (Doppler shifted)
- **Two hydrodynamic modes** (disappear for $M_0 \to 0$)
  - One with **positive energy** $k_S$
  - One with **negative energy** $k_N$
- One boundary layer mode $k_B$ (disappear for $\delta \to 0$)
Lined ducts with mean flow

Supersonic flows $M_0 > c_{\text{eff}}$

- Hydrodynamic modes disappear
- Two acoustic modes \textbf{co-moving} with flow
Impedance change $\rightarrow$ transsonic flow

- $M_0 > c_{\text{eff}} \rightarrow$ wave trapping
- Acoustic modes cannot propagate against the flow
- **Acoustic analogue of a black hole**

[Unruh ’81 [...], Auregan, Fromholz, Michel, Pagneux, Parentani 15’]
Impedance change $\rightarrow$ transsonic flow

Important properties of transsonic flows:

**Couple very different wavelengths**
(due to Doppler effect)
### Effective transsonic flows

#### Supersonic flow
- $\gamma \rightarrow k_B$
- Evanescent $\rightarrow$
- $0 \rightarrow k_N$
- $0 \rightarrow k_{A+}$

#### Subsonic flow
- $k_N \rightarrow \beta$
- $k_{A+} \rightarrow R$
- $k_S \rightarrow \alpha$
- $k_B \rightarrow 0$
- $k_{A-} \rightarrow 1$

- Impedance change $\Rightarrow$ 4 × 4 scattering matrix
- Energy conservation

\[ |\alpha|^2 - |\beta|^2 + |\gamma|^2 + |R|^2 = 1 \]

- Possibility of **amplification**
  $\rightarrow$ analogue of the **Hawking radiation** of black holes
Double transsonic flows

Supersonic flow

\[ k_B \quad \rightarrow \quad k_N \quad \rightarrow \quad k_{A+} \]

Subsonic flow

\[ k_B \quad \rightarrow \quad k_{A-} \quad \rightarrow \quad k_N \quad \rightarrow \quad k_{A+} \quad \rightarrow \quad k_S \]

Supersonic flow

\[ k_B \quad \rightarrow \quad k_{Rev} \quad \rightarrow \quad k_N \quad \rightarrow \quad k_{A+} \]
Double transsonic flows

Supersonic flow

$\dot{k}_B$

Subsonic flow

$\dot{k}_B$

Energy exchange

$\dot{k}_{A-}$

Resonant cavity

$\dot{k}_N$

Energy exchange

$\dot{k}_S$

Energy exchange

$\dot{k}_{A+}$

$\dot{k}_B$

$E_N < 0$

$E_{cav.} > 0$

$E_{A+,B} > 0$
Double transsonic flows

- Described by complex eigen-frequencies $\omega \in \mathbb{C}$
- Eigen-value problem
Double transsonic flows

Eigen-value problem

- Boundary conditions
  - $\text{Im}(\omega) > 0$: decaying for $x \to \pm \infty$
    - unstable solutions
  - $\text{Im}(\omega) < 0$: analytic continuation
    - resonances

  → equivalent to outgoing boundary conditions

- Spectrum symmetry $\text{Re}(\omega) \to -\text{Re}(\omega)$
Double transsonic flows

Eigen-value problem

- Boundary conditions
  - $\text{Im}(\omega) > 0$: decaying for $x \to \pm \infty$
    $\rightarrow$ unstable solutions
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  $\rightarrow$ equivalent to outgoing boundary conditions

- Spectrum symmetry $\text{Re}(\omega) \rightarrow -\text{Re}(\omega)$

- How does the spectrum change with external parameters?
  $\rightarrow$ Two main properties

Eigen-value problem
Slow sound laser

Parameters $b_O = 15, \ b_C = 6, \ M = 0.3, \ \delta = 0.002$

- $L$ varies from 0.2 to 3.6

Two types of unstable modes:
- Static $\text{Re}(\omega) = 0$
- Dynamic $\text{Re}(\omega) \neq 0$
Slow sound laser

Mode profiles - Subsonic region in grey

- **Subwavelength** instability
- Governed by hydrodynamic wavelengths:
  Unstable mode if \( k_S L \approx \pi/2 \) with \( k_S \) hydrodynamic mode
Conclusion

Slow sound laser:
- Double transsonic flows
- Can be **static** or **dynamic**


Analogous to “black hole laser”
→ leads to rich nonlinear phenomenology
  - Undular bore? (static)
  - Dispersive shock waves? (dynamic)
  - Emission of solitons? (dynamic)
Conclusion

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Thank you.