# Slow sound laser in lined flow ducts

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**CEAS-ASC Workshop:**
**New Materials for Applications in Aeroacoustics**

From: Laboratoire d'Acoustique de l'Université du Mans

# **Analogy** [Unruh 1981]

Waves in a **spacetime**

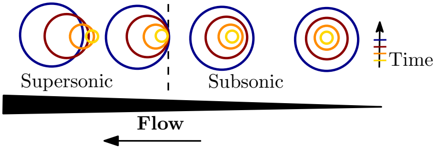
$$\partial_\mu \sqrt{|g|} g^{\mu\nu} \partial_\nu \phi = 0$$

Picture of a Black Hole

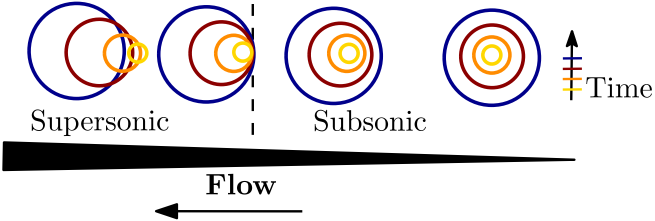
(From: Event Horizon Telescope '19)

**Acoustic** waves with mean flow

$$(\partial_t + \nabla \cdot \mathbf{v}) \frac{\rho}{c^2} (\partial_t + \mathbf{v} \cdot \nabla)\phi - \nabla \cdot \rho \nabla \phi = 0$$



What happens when flow becomes faster than waves?




- Wave trapping
  → acoustic analogue of a black hole: *Dumb hole*
- Exotic wave effects around black holes could be reproduced

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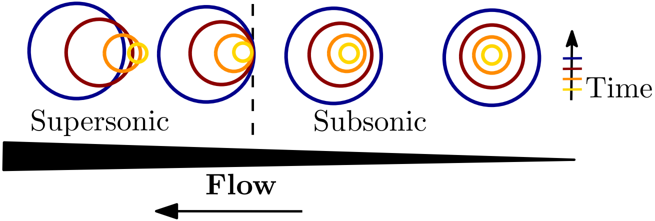


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**"Slow sound": an opportunity to realize dumb holes?**

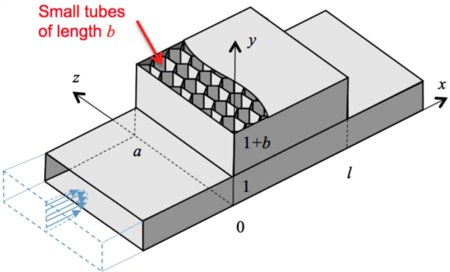
## Speed of sound:

$$c_0 = \sqrt{\frac{K}{\rho_0}}$$

$K$: isentropic bulk modulus
$\rho_0$: density

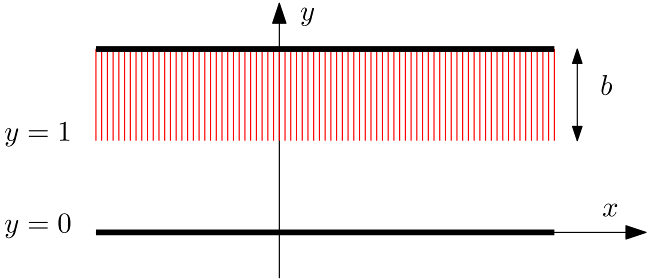
## Slow sound:

- Tubes mounted flush
  $\rightarrow$ **Metamaterial like**
- **Lower the effective stiffness**
- Long wavelengths:
  $c_{\text{eff}} < c_0$




Waveguide with impedance treatment

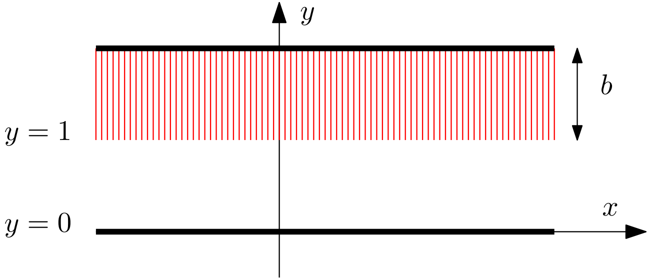
## Wave propagation in lined ducts




- Units where duct height $H = 1$ and $c_0 = 1$
- Potential perturbations $\mathbf{v} = \nabla\phi$
- Inside the duct

$$\partial_t^2 \phi - \Delta\phi = 0.$$

# Wave propagation in lined ducts



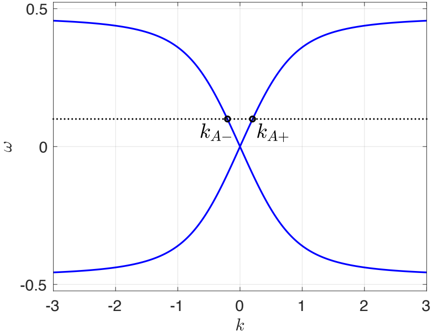

**Boundaries:**

- $y = 0$ hard wall: $\partial_y \phi = 0$
- $y = 1$ **impedance wall**: $p = Z \partial_y \phi$
- **Complex** impedance $Z$. Tube model:

$$Z(\omega) = \frac{i}{\sigma \tan(b\omega)} + \Gamma$$

  - Dissipation $\Gamma$ neglected
  - **Low frequencies** $\omega \ll \pi/2b$ (subresonance)

# Wave propagation in lined ducts

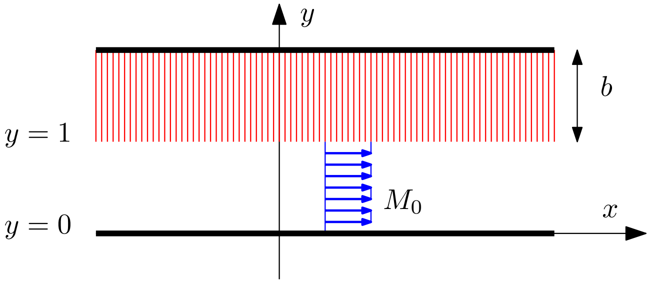



- Fixed frequency $\omega = 2\pi f$
- **Two** acoustic modes
- $\omega \ll \pi/2b$:

$$\omega^2 = \frac{k^2}{1+b}$$

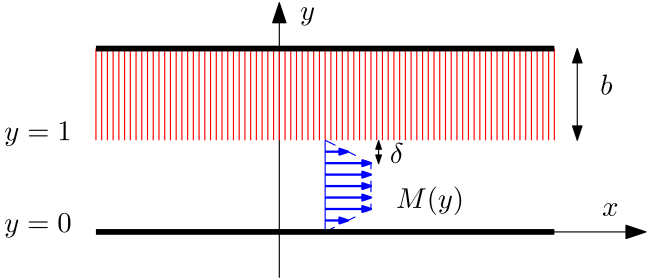
  Hence $c_{\text{eff}} = 1/\sqrt{1+b}$
- Increase $\omega \to$ **dispersion**

# Lined ducts with mean flow




- Mean Mach number $M_0 = U_0/c_0$
- **Two** types of flow:
  - **Subsonic:** $M_0\sqrt{1+b} < 1$
  - **Supersonic:** $M_0\sqrt{1+b} > 1$
- Possible for $M_0 < 1$
- What is the dispersion relation?

## Lined ducts with mean flow

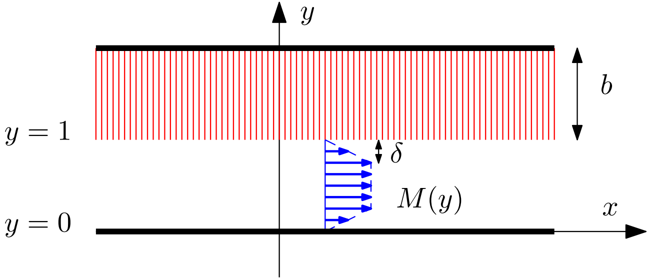



- Inside the duct

$$(\partial_t + M\partial_x)^2\phi - \Delta\phi = 0.$$

- **Hard problem:** vorticity modes, inhomogeneities, etc.
- $M \approx M_0$ constant except in **boundary layer** $1-\delta \lesssim y < 1$

## Lined ducts with mean flow




- Asymptotic approach: **effective boundary condition**

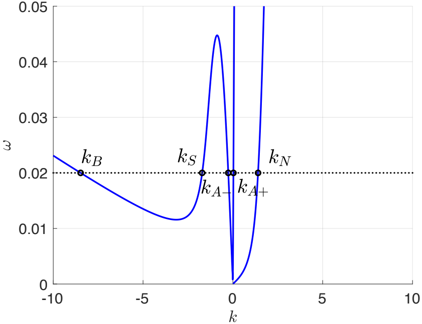
  [Brambley '13, AC, Aurégan, Pagneux '19]

- Our boundary condition
  - pressure $p$ is continuous
  - displacement $\eta$ is continuous
  - **But** effective compliance

$$C_{\text{eff}}(\omega) = \frac{\eta}{p} = C_0(\omega) + \delta\, C_1(\omega)$$

  - $\delta = 0$ is Ingard-Myers boundary conditions

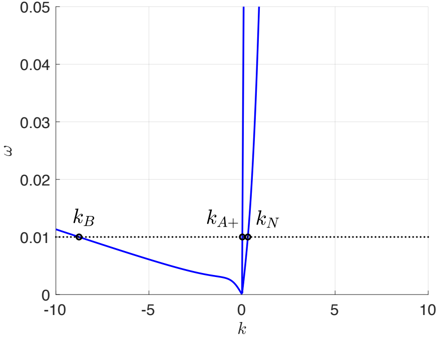
## **Lined ducts** with **mean flow**




**Subsonic** flows $M_0 < c_{\text{eff}}$

- Two acoustic modes (Doppler shifted)
- **Two hydrodynamic modes** (disappear for $M_0 \to 0$)
  - One with **positive** energy $k_S$
  - One with **negative energy** $k_N$
- **One boundary layer mode** $k_B$ (disappear for $\delta \to 0$)

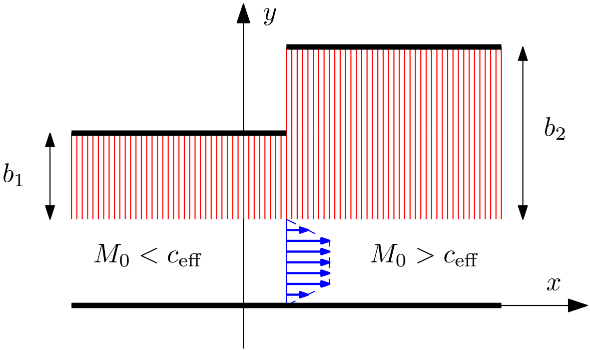
## Lined ducts with mean flow




**Supersonic** flows $M_0 > c_{\text{eff}}$

- **Hydrodynamic modes** disappear
- Two acoustic modes **co-moving** with flow

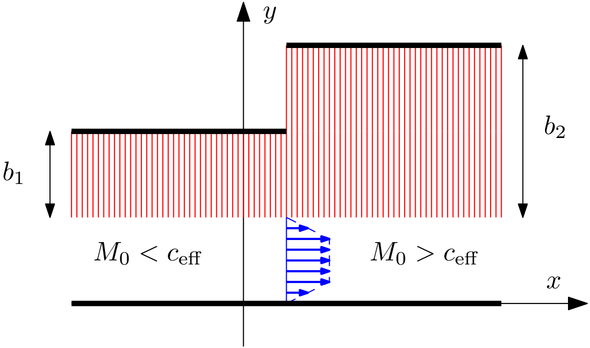
## Impedance change → transsonic flow




- $M_0 > c_{\text{eff}} \rightarrow$ **wave trapping**
- Acoustic modes cannot propagate against the flow
- **Acoustic analogue of a black hole**

  [Unruh '81 [...], Auregan, Fromholz, Michel, Pagneux, Parentani 15']

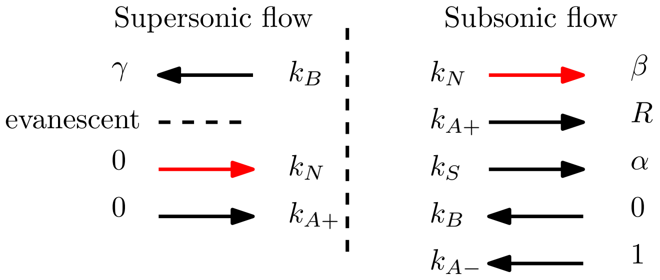
## Impedance change → transsonic flow




Important properties of transsonic flows:

**Couple very different wavelengths**
(due to Doppler effect)

## Effective transsonic flows

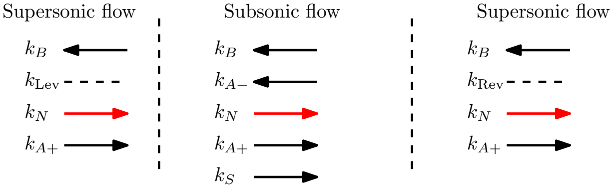



- Impedance change $\Rightarrow 4 \times 4$ scattering matrix
- Energy conservation

$$|\alpha|^2 - |\beta|^{\mathbf{2}} + |\gamma|^2 + |R|^2 = 1$$

- Possibility of **amplification**
  $\rightarrow$ analogue of the **Hawking radiation** of black holes

# Double transsonic flows



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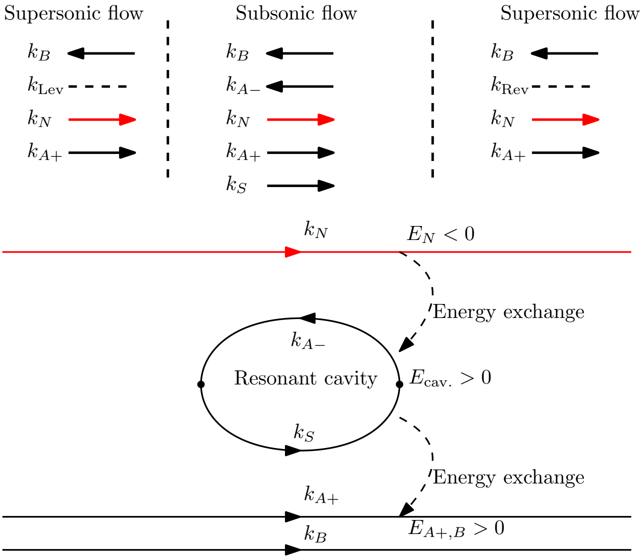

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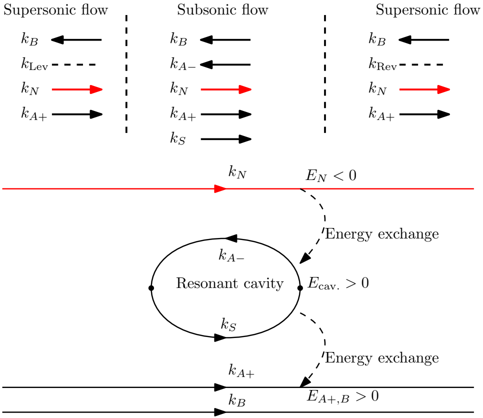
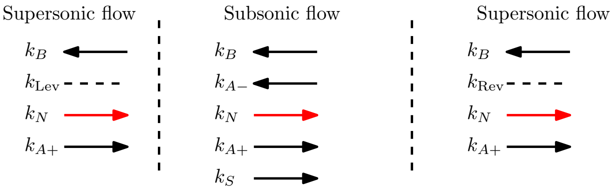



- Described by **complex eigen-frequencies** $\omega \in \mathbb{C}$
- **Eigen-value problem**

## Double transsonic flows

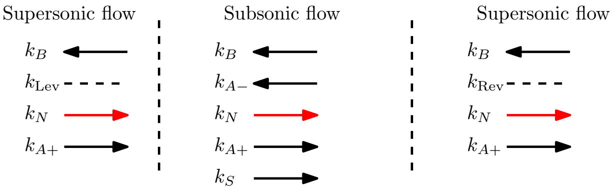



### Eigen-value problem

- Boundary conditions
  - $\text{Im}(\omega) > 0$: decaying for $x \to \pm\infty$
    $\to$ **unstable solutions**
  - $\text{Im}(\omega) < 0$: analytic continuation
    $\to$ **resonances**

  $\to$ equivalent to outgoing boundary conditions
- Spectrum symmetry $\text{Re}(\omega) \to -\text{Re}(\omega)$

## Double transsonic flows




**Eigen-value problem**

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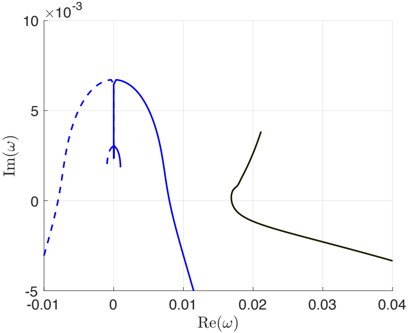
    $\rightarrow$ **resonances**

  $\rightarrow$ equivalent to outgoing boundary conditions
- Spectrum symmetry $\mathrm{Re}(\omega) \to -\mathrm{Re}(\omega)$
- How does the spectrum change with external parameters?
  $\rightarrow$ **Two main properties**

# Slow sound laser



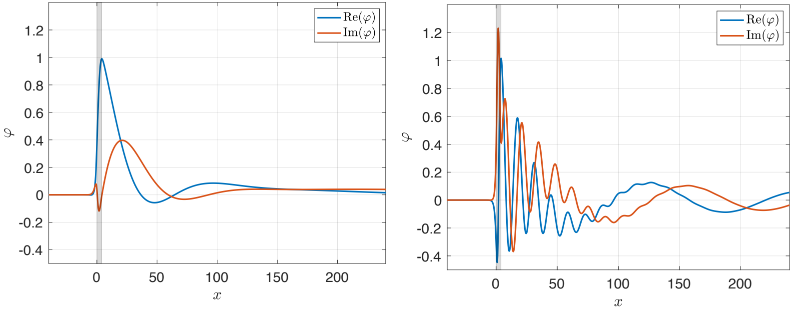

Parameters $b_O = 15$, $b_C = 6$, $M = 0.3$, $\delta = 0.002$

- $L$ **varies** from 0.2 to 3.6

**Two types of unstable modes:**

- Static $\mathrm{Re}(\omega) = 0$
- Dynamic $\mathrm{Re}(\omega) \neq 0$

# Slow sound laser




Mode profiles - Subsonic region in grey

- **Subwavelength** instability
- Governed by hydrodynamic wavelengths:
  Unstable mode if $k_S L \approx \pi/2$ with $k_S$ hydrodynamic mode

## Conclusion

**Slow sound laser:**

- Double transsonic flows
- Can be **static** or **dynamic**

[AC, Aurégan, Pagneux, JASA 2019, *arXiv:1904.03079*]

**Analogous to "black hole laser"**
**→ leads to rich nonlinear phenomenology**

- Undular bore? (static)
- Dispersive shock waves? (dynamic)
- Emission of solitons? (dynamic)

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# Thank you.