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An integrated approach to the theoretical and numerical modelling of metamaterials for aeroacoustics

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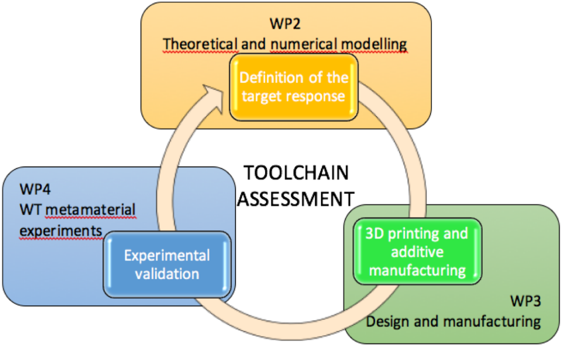
Summary

Integrated design of a *metacontinuum* (MC) for aeroacoustic applications. AERIALIST WP2

1. Model a generic aeroacoustic metacontinuum
2. Optimisation procedure to match the MC properties to the target response
3. Integration with the Inverse Estimation Method

Work is in progress. Only partial preliminary results are presented here and in KTH presentation.

AERIALIST Loop



AERIALIST WP2

Consolidate theory for aeroacoustic metamaterials

- Convective MM design
- Acoustic analogy for metacontinua
- Modelling of visco-thermal losses
- Modelling for anisotropic elasticity
- Modelling of meta surfaces

Adapt and enhance numerical methods for MM simulations

- Numerical maps for ATA
- Boundary-Field Integral Equations
- Extended BEM and FEM models
- Micro-to-Macro elasticity models
- Generalised BEM/FEM non-local boundary conditions

Numerical optimisation to identify a MM from target response

- Inverse Estimation Method
- Optimised metacontinuum to match a target response

General *metafluid* model (Norris 2009)

A MetaContinuum (MC) such that the Cauchy stress tensor has the form $\boldsymbol{\sigma} = -p\,\mathbf{Q}$ is a *MetaFluid* (MF).

With the only constraint is $\nabla \cdot \mathbf{Q} = 0$, an *acoustic* disturbance propagates according to

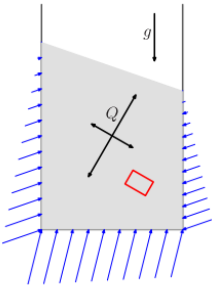
$$-\frac{1}{c_{\text{ref}}^2}\frac{\partial^2 p}{\partial t^2} + \mathcal{K}\,\mathbf{Q} : \nabla\left(\varrho^{-1}\,\mathbf{Q}\,\nabla p\right) = 0$$

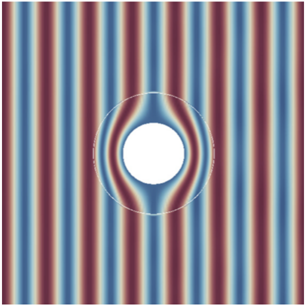
The Hooke's tensor for such a metacontinuum is

$$\mathbf{C} = \mathcal{K}\,\mathbf{Q} \otimes \mathbf{Q}$$

General *metafluid* model

A lot of funny things can be done...



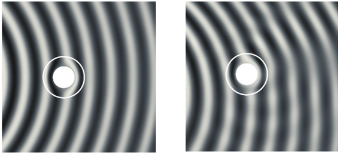


Anisotropic **Q** for non-horizontal free surface (Norris 2009).

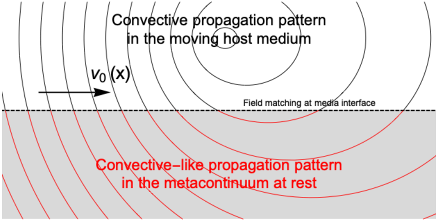
Anisotropic $\hat{\varrho}$ for scattering cancellation

Convective correction

When operating in a flow, the response of a statically-designed MC deteriorates...

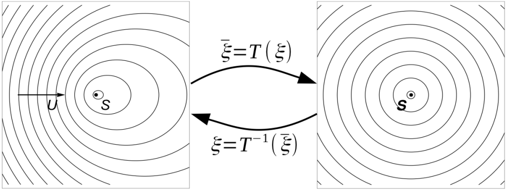


Its properties must be corrected to cope with aerodynamic convection



Convective correction

Space-time coordinate transformation can do the job (Lorentz for uniform free stream or Taylor for low–Mach non–uniform flow)



Problem recast in space-time form.

$$\boldsymbol{\xi} \equiv (\xi_0, \xi_1, \xi_2, \xi_3) = (c_{\text{ref}} t, x_1, x_2, x_3)$$
$$\boldsymbol{\partial} \equiv (\partial_0, \partial_1, \partial_2, \partial_3) = (\partial_t / c_{\text{ref}}, \partial_i)$$

For details, see Iemma and Palma, JSV v. 443, 2019 and paper in preparation.

Convective correction

MC wave equation rewritten as ($c_{\text{ref}} = 1$)

$$\partial \cdot \left(\mathbf{g}^{-1}\, \partial p\right) = 0, \quad \textit{with} \quad \mathbf{g} = \begin{pmatrix} -1 & \vdots & 0 \\ \cdots & \cdot & \cdots \\ 0 & \vdots & \mathbf{Q}^{-1}\,\hat{\varrho}\,\mathbf{Q}^{-1} \end{pmatrix}$$

$\mathbf{g}$ is the metrics of the MC wave equation. If

$$\mathbf{g} = \begin{pmatrix} -1 & \vdots & 0 \\ \cdots & \cdot & \cdots \\ 0 & \vdots & \mathbf{I} \end{pmatrix}$$

we obtain the standard wave equation in a medium at rest.

Convective correction

Convective metrics obtained by applying ST transformation to $\mathbf{g}$

$$\hat{\mathbf{g}} = \mathbf{T}\,\mathbf{g}\,\mathbf{T}^T$$

where $\mathbf{T}$ is the linear map $\hat{\boldsymbol{\xi}} = \mathbf{T}\,\boldsymbol{\xi}$.
For a Lorentz boost associated to uniform $\mathbf{v}$

$$\mathbf{T} = \begin{pmatrix} 1/\gamma & \vdots & -\mathbf{m}^T/\beta \\ \cdots & \cdot & \cdots \\ -\mathbf{m}/\beta & \vdots & \mathbf{I} + \frac{(1-\beta)}{\beta\, m^2}\mathbf{m}\otimes\mathbf{m} \end{pmatrix}.$$

with $\beta = \sqrt{1 - m^2}$, and $\mathbf{m} = \mathbf{v}/c_{\text{ref}}$.

Analogy for the convective MF

The equation for the corrected properties is

$$\hat{\mathbf{g}} = \left(\hat{\varrho}^{-1}\,\hat{\mathbf{Q}}\right)^{-1} \Rightarrow \hat{\mathbf{C}} = \hat{\mathcal{K}}\,\hat{\mathbf{Q}} \otimes \hat{\mathbf{Q}}$$

Note that any couple $(\hat{\varrho}, \mathbf{Q})$ satisfying the equation is feasible.
The corrected equation is written as

$$\nabla^2 p - \frac{1}{c_{\mathrm{ref}}^2}\frac{\partial^2 p}{\partial t^2} = \nabla \cdot \hat{\mathbf{q}} + \hat{\sigma}$$

with

$$\hat{\mathbf{q}} = \left(\mathbf{I} - \hat{\mathbf{Q}}\hat{\varrho}^{-1}\hat{\mathbf{Q}}\right)\,\nabla p + \hat{\mathbf{Q}}\hat{\varrho}^{-1}\hat{\mathbf{Q}}\,\nabla A\,p$$

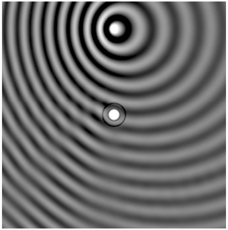
$$\hat{\sigma} = \left(\frac{1}{\mathcal{K}_{ref}} - \frac{1}{\hat{\mathcal{K}}}\right)\frac{\partial p}{\partial t}$$

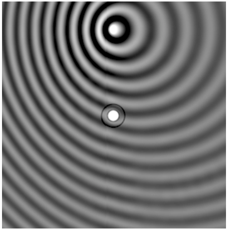
Easily integrable in BEM and FEM codes.

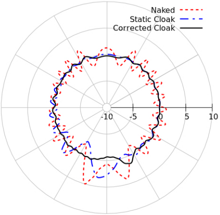
BEM and FEM implementation

The MM Boundary–Field Integral Equation is

$$E_c(\mathbf{y})\,\tilde{p}(\mathbf{y}) = \oint_{\partial\Omega_c} \left\{ G(\mathbf{x},\mathbf{y}) \left[\frac{\partial \tilde{p}}{\partial n} - \mathbf{q}(\mathbf{x}) \cdot \mathbf{n} \right] - \tilde{p}\, \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n} \right\} \mathrm{d}\Gamma(\mathbf{x})$$
$$- \int_{\Omega_c} \mathbf{q}(\mathbf{x}) \cdot \nabla G(\mathbf{x},\mathbf{y})\, \mathrm{d}\Omega(\mathbf{x}) + \int_{\Omega_c} G(\mathbf{x},\mathbf{y})\, \sigma(\mathbf{x})\, \mathrm{d}\Omega(\mathbf{x})$$



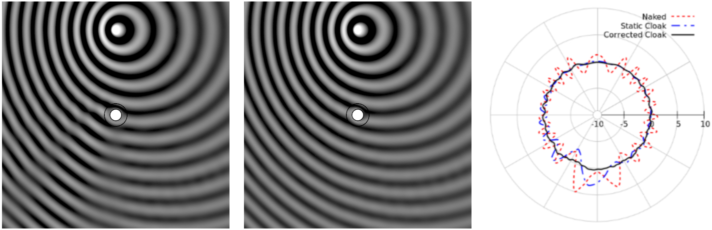




(Iemma and Palma, JSV v. 443, 2019 plus paper in preparation...

BEM and FEM implementation

Similar results are obtained integrating Taylor transformation for low–Mach non–uniform flows into FEM code COMSOL



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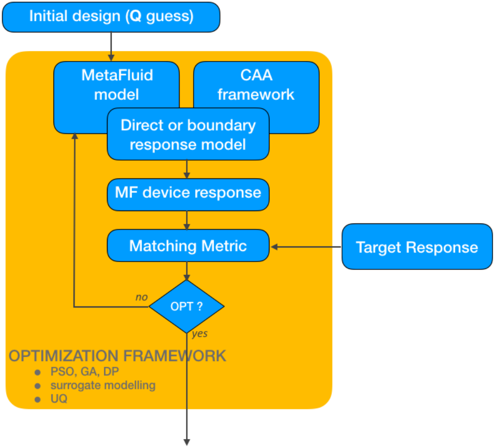
Metacontinuum optimization

- Objective function is a measure of the matching of the target and simulated responses. Typically, the L_p–norm of the Δ response.
- Optimisation variables are the components of $\mathbf{g}$ or $\hat{\mathbf{g}}$ (Optimise and then convect or optimise the convective ? T.B.D.)
- Geometric and manufacturing constraints.

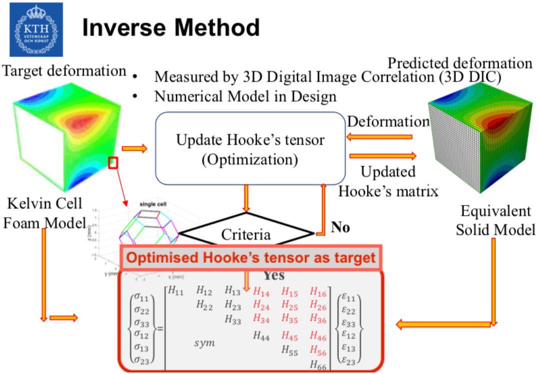
$$\text{Min } J(g_{\mu\nu}) = \left(\int_{\mathcal{D}} |\Delta\mathcal{R}(g_{\mu\nu})|\right)^{1/p}$$

$$\text{with } h_k(g_{\mu\nu}) \leq 0, \quad k = 1, \ldots, N$$



The *Integration with Inverse Optimization Method*

KTH developed an inverse method to characterise a continuum matching a target response (details in Mao and Göransson presentation).



Concluding Remarks

- An integrated approach to the design of aerocoustic metamaterial suitable to operate in a flow has been presented
- The partial results obtained so far demonstrate the effectiveness of each single part of the formulation
- The comprehensive formulation is currently being implemented in AERIALIST to be assessed during the last part of the project