An inverse method for design and characterisation of acoustic materials

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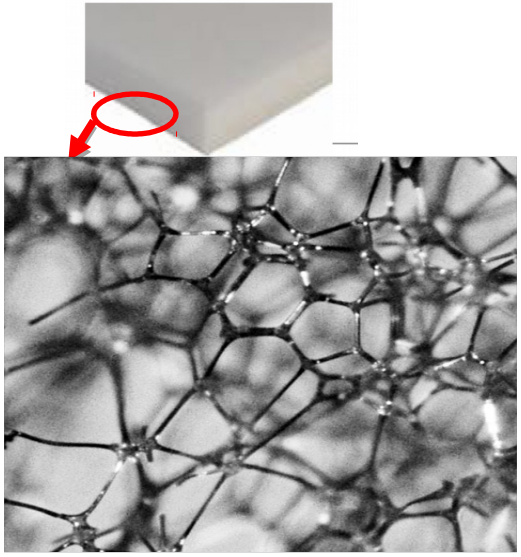
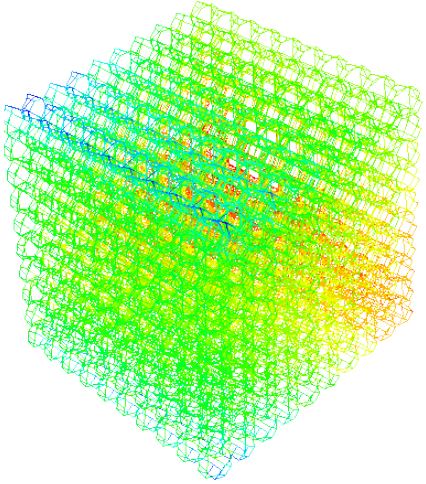
The Center for ECO2 Vehicle Design

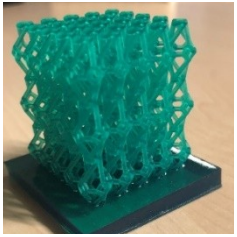
Acoustic Material: Foams

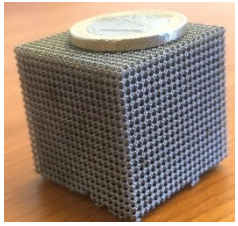
Melamine Foam



Designed & Printed Foam

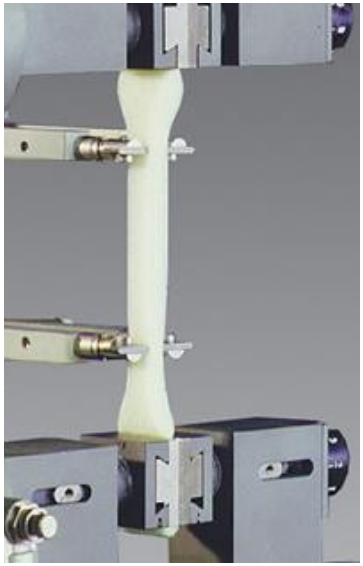






Prints by TCD™

Material Properties Characterisation

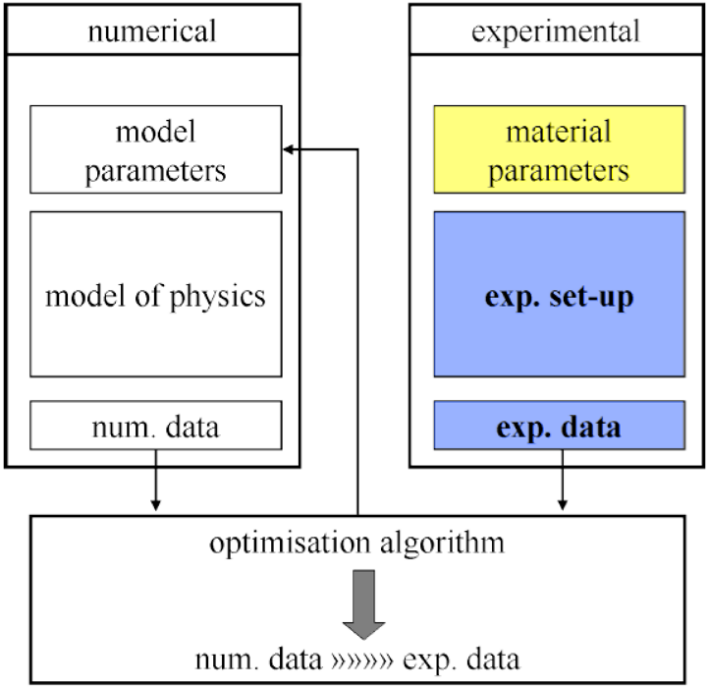


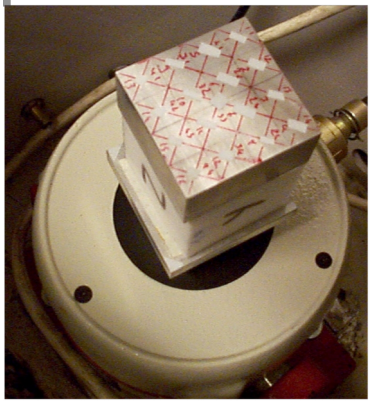
- Anisotropic ?

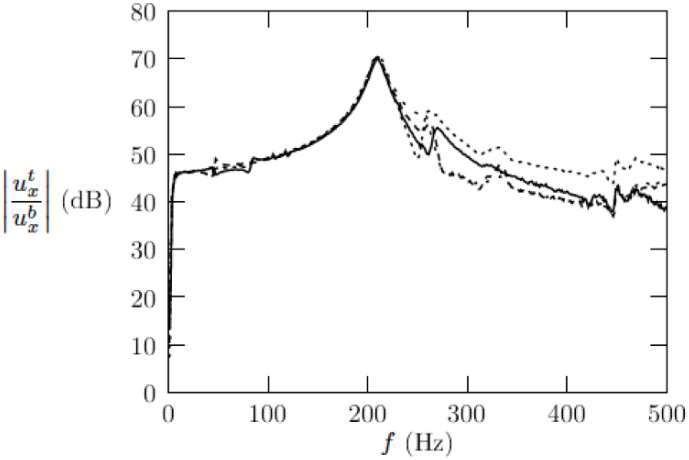
$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}
=
\begin{bmatrix}
H_{11} & H_{12} & H_{13} & \overset{?}{H_{14}} & \overset{?}{H_{15}} & \overset{?}{H_{16}} \\
 & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\
 & & H_{33} & H_{34} & H_{35} & H_{36} \\
 & & & H_{44} & H_{45} & H_{46} \\
 & sym & & & H_{55} & H_{56} \\
 & & & & & H_{66}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix}
$$

- Stiffness
- Shear modulus
- Poisson's ratio
- Iso/Orthotropic

Melamine - Anisotropic-anelasticity





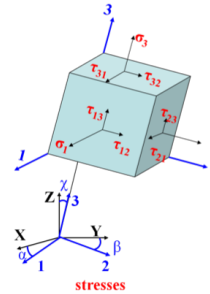


Measurements: Christophe Van der Kelen

Elastic moduli from dynamic estimation

$$\mathbf{C} = \begin{bmatrix} 760 & 340 & 5 & 37 & 20 & -31 \\ & 430 & 120 & 11 & 9 & -16 \\ & & 220 & 12 & 2 & 14 \\ & & & 100 & -3 & -2 \\ & symm & & & 120 & -4 \\ & & & & & 100 \end{bmatrix} \{kPa\}$$

Rotations identified
-7°,0°,8°



$E_1 = 448$ kPa $\quad \nu_{21} = 0.445$

$E_2 = 211$ kPa $\quad \nu_{13} = -0.514$

$E_3 = 170$ kPa $\quad \nu_{32} = 0.433$

$G_{23} = 104$ kPa

$G_{31} = 124$ kPa

$G_{12} = 101$ kPa

$$H_{ij}(\omega) = C_{ij} + \frac{B_{ij}(i\omega/\beta_{ij})^{\alpha}}{1+(i\omega/\beta_{ij})^{\alpha}}$$

$$B_{ij} = bC_{ij}$$

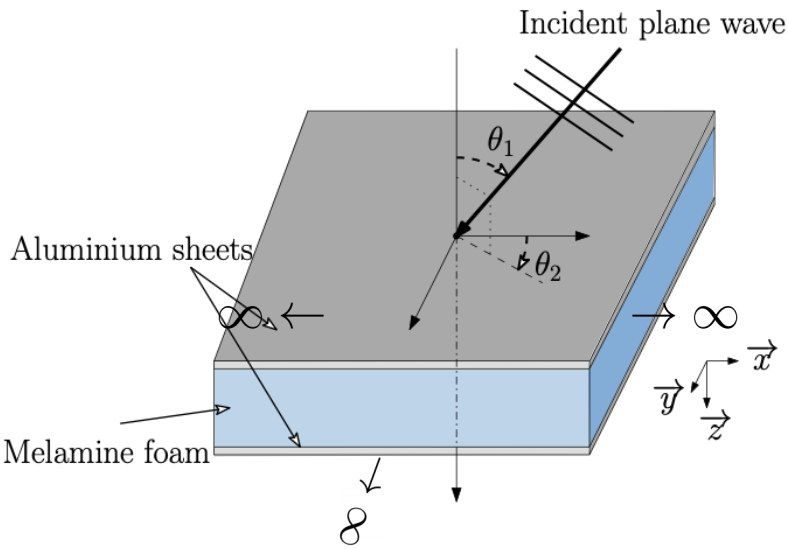
$\beta = 813$ krad/s

$\alpha = 0.333$

$b = 0.296$

Cuenca J, Van der Kelen C and Göransson P. A general methodology for inverse estimation of the elastic and anelastic properties of anisotropic open-cell porous materials. J Appl Physics **115**, 2014.

Geometry and material



Aluminium - 1mm
[isotropic]

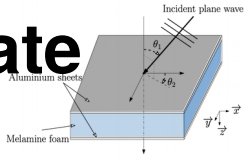
Mélamine foam - 88mm
[anisotropic | anelastic]

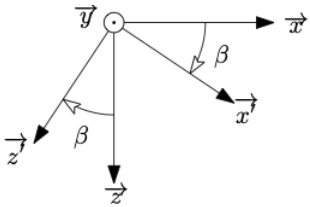
Frequency domain
[100 Hz – 5 kHz]

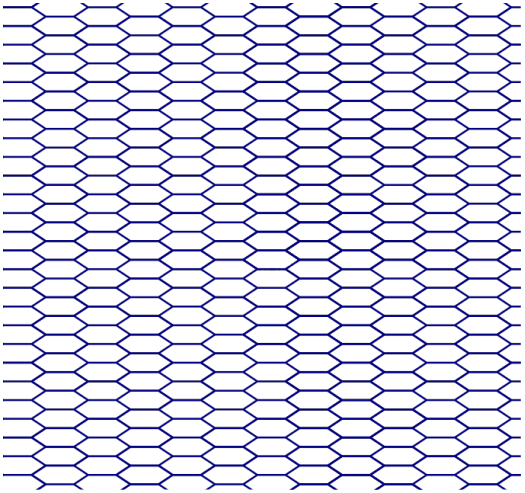
Normal Incidence

Parra Martinez, J. P., Dazel, O., Göransson, P., Cuenca, J., Acoustic analysis of anisotropic poroelastic multilayered systems. Journal of Applied Physics, 119, 084907 (2016)

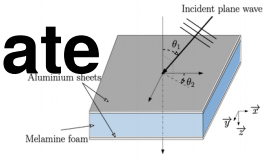
Rotation of material coordinate system

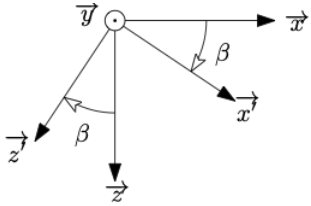






Rotation of material coordinate system

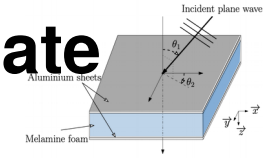


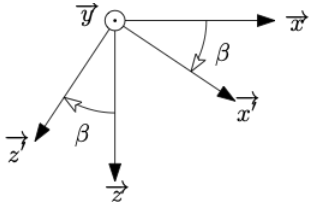


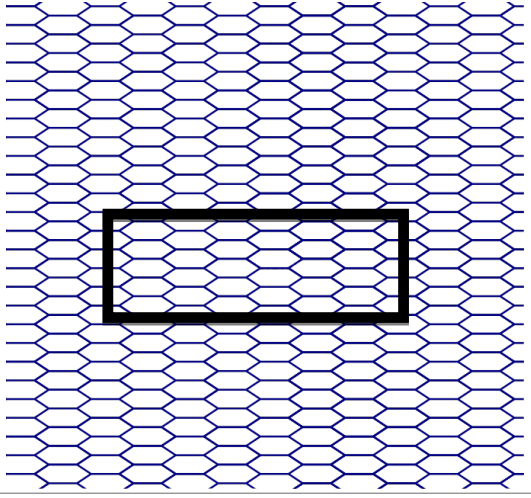
Isotropic equivalent*

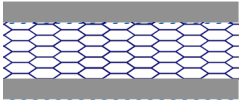
*Norris, J. Mech. Mat. Struct. 2(2005)

Rotation of material coordinate system



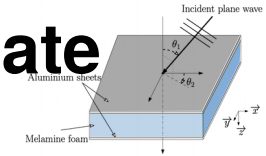


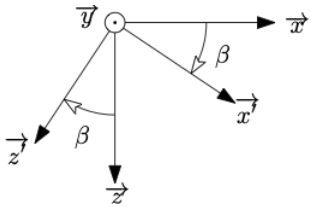


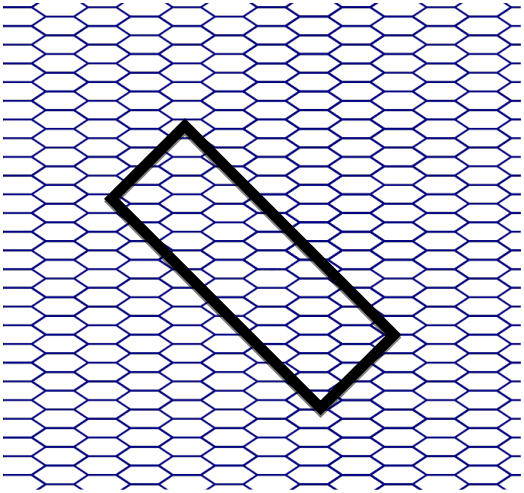
$\infty \leftarrow$  $\rightarrow \infty$

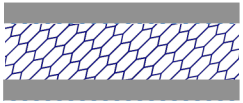
$\beta = 0$ rad

Rotation of material coordinate system



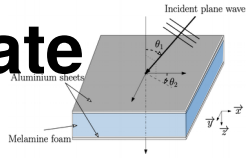


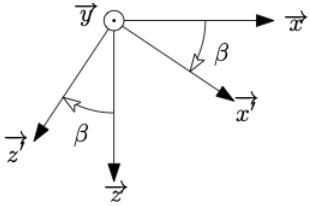


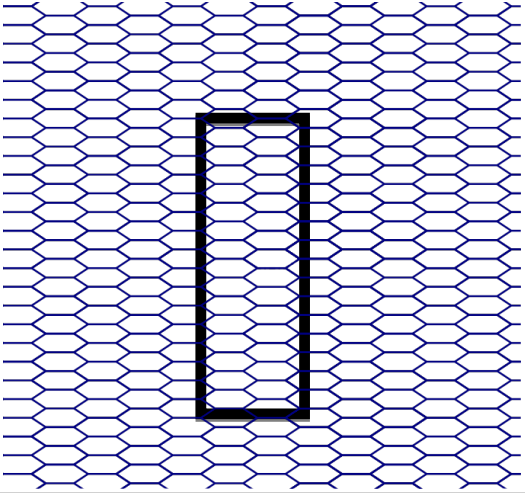
$\infty \leftarrow$  $\rightarrow \infty$

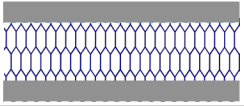
$$\beta = \frac{\pi}{4}\ \mathrm{rad}$$

Rotation of material coordinate system



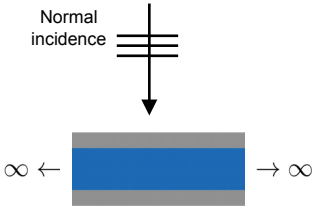




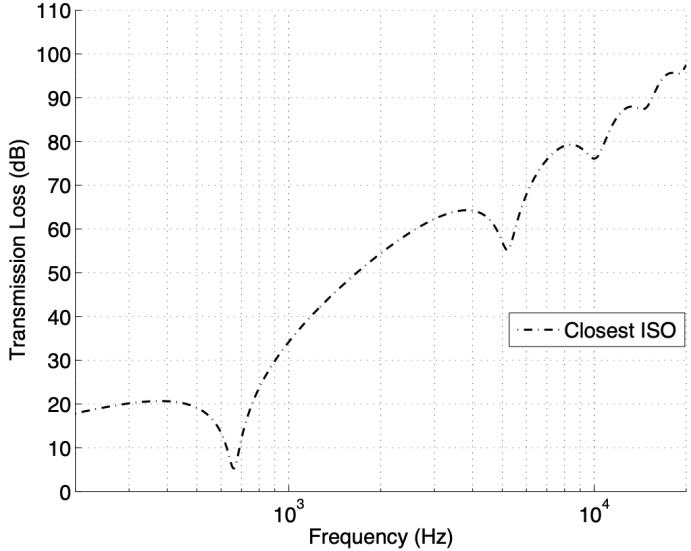
$\infty \leftarrow$  $\rightarrow \infty$

$$\beta = \frac{\pi}{2} \text{ rad}$$

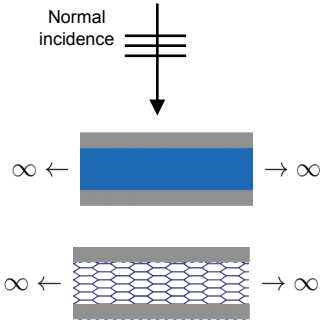
Transmission loss



Isotropic
equivalen
t*

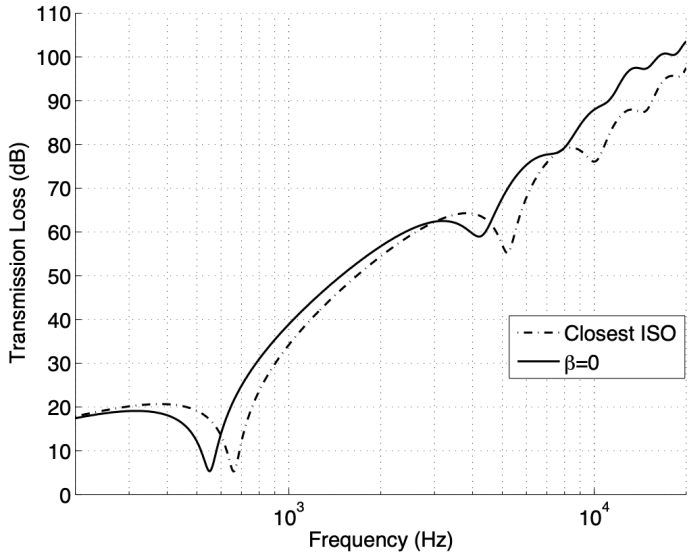


Transmission loss

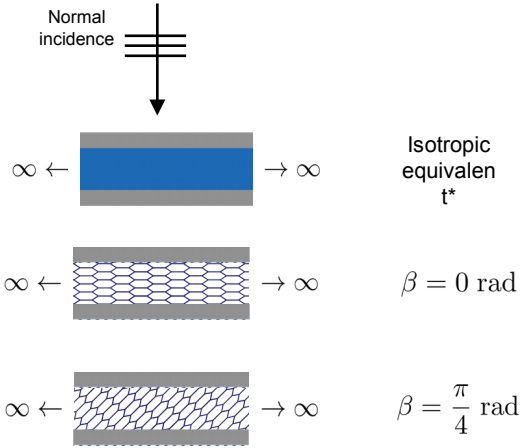


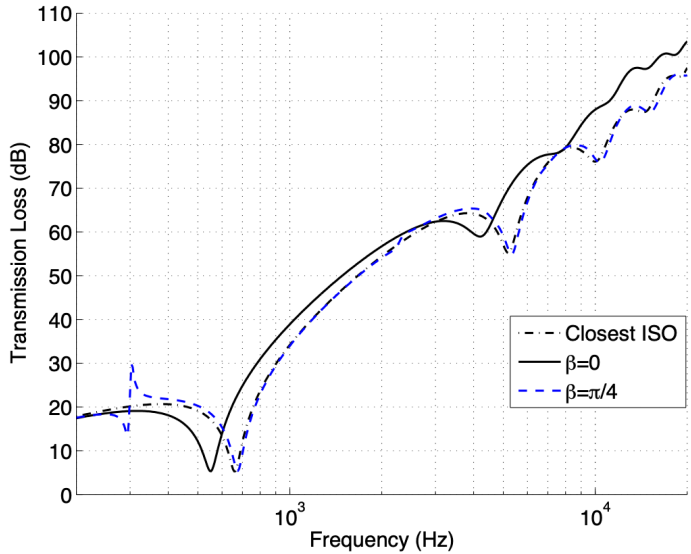
Isotropic
equivalen
t*

$\beta = 0$ rad

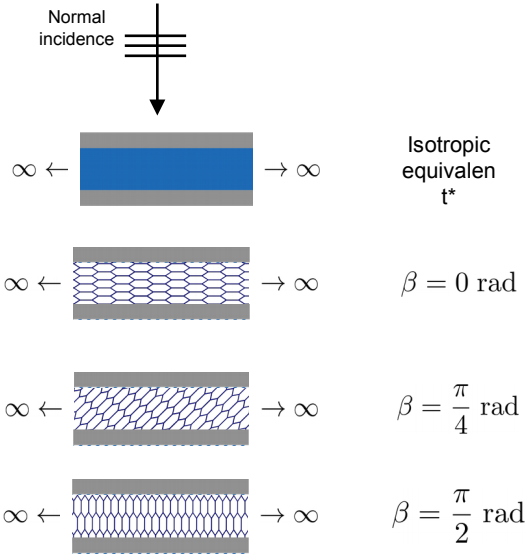


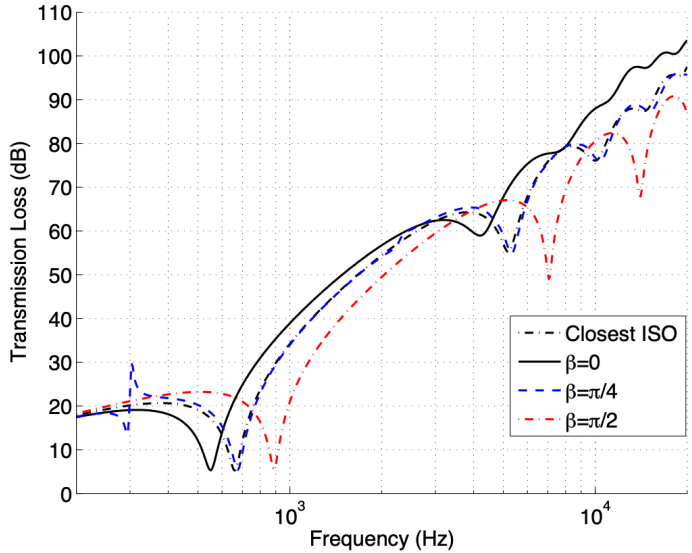
Transmission loss



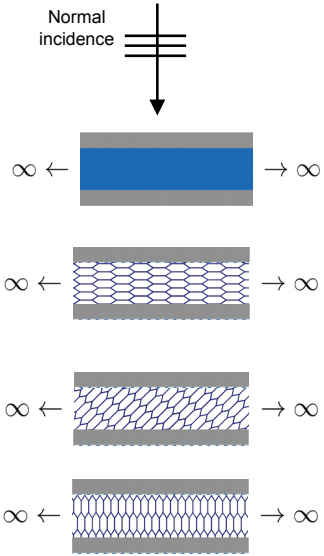


Transmission loss





Transmission loss

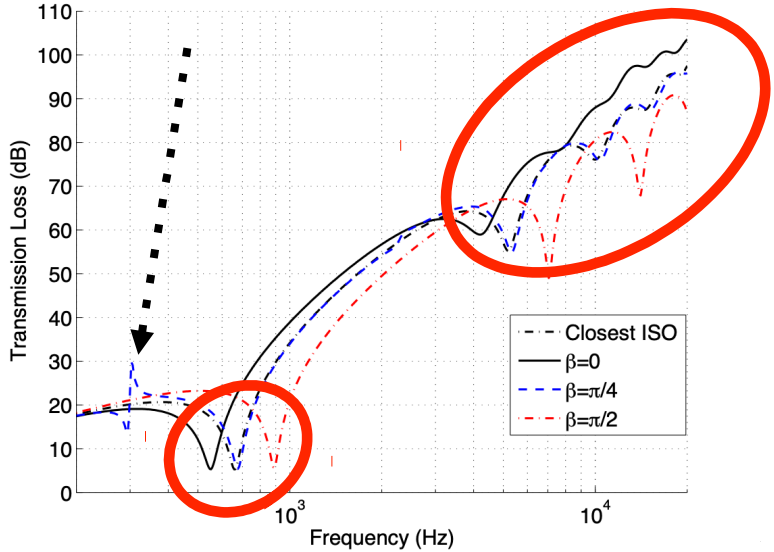


Isotropic
equivalen
t*

$\beta = 0$ rad

$\beta = \frac{\pi}{4}$ rad

$\beta = \frac{\pi}{2}$ rad

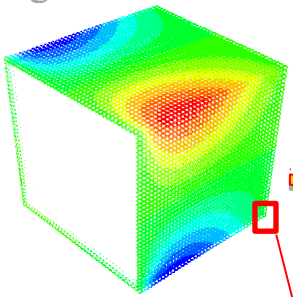


Inverse Method

Target deformation

Measured by 3D DIC
Numerical Model in Design

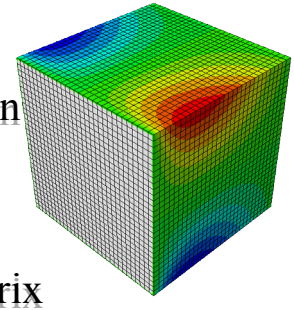
Predicted deformation



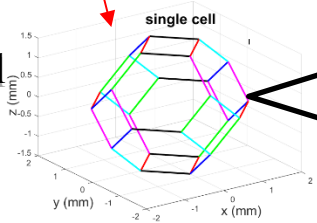
Update Hooke's tensor
(Optimization)

Deformation

Updated
Hooke's matrix



Kelvin Cell
Foam Model



Criteria

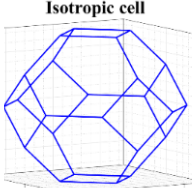
No

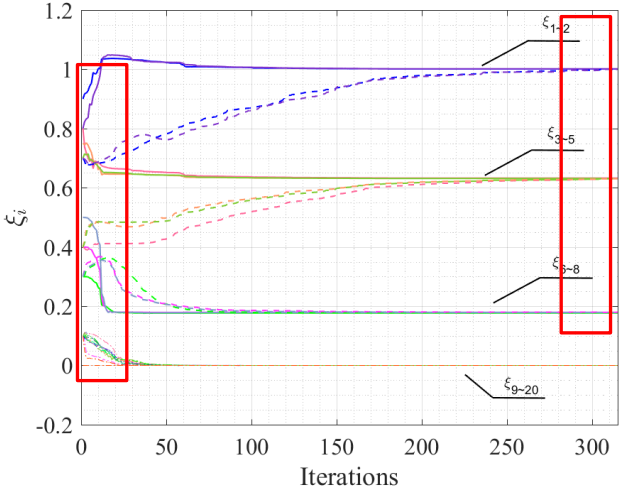
Yes

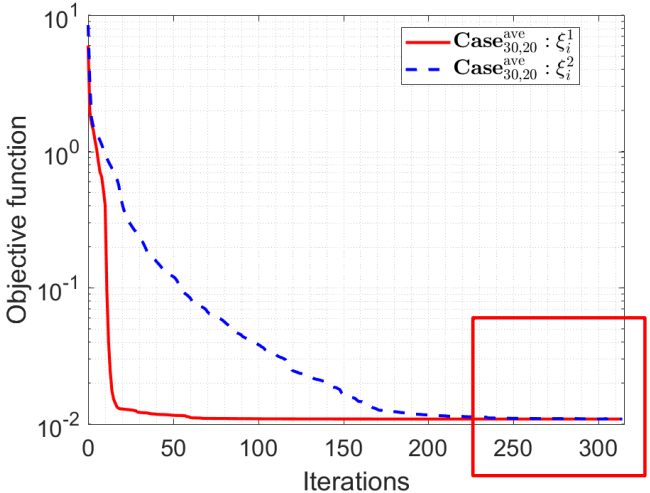
Equivalent
Solid Model

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ & & & H_{44} & H_{45} & H_{46} \\ & sym & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix}$$

Convergence of the Method
Isotropic cell





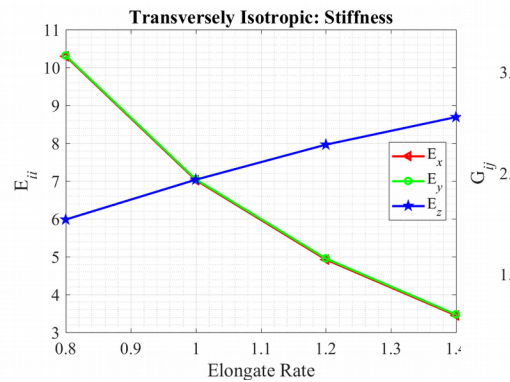


$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ & & & H_{44} & H_{45} & H_{46} \\ & sym & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{bmatrix} = \begin{bmatrix} 14.6 & 9.6 & 9.6 & 0 & 0 & 0 \\ & 14.6 & 9.6 & 0 & 0 & 0 \\ & & 14.6 & 0 & 0 & 0 \\ & & & 2.6 & 0 & 0 \\ & sym & & & 2.6 & 0 \\ & & & & & 2.6 \end{bmatrix} \text{MPa}$$

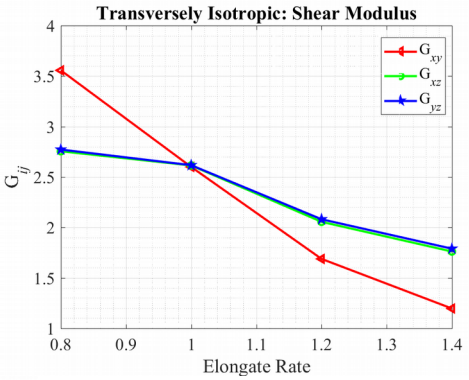
$$\mathbf{H} = \text{MPa}$$

Elongated cells

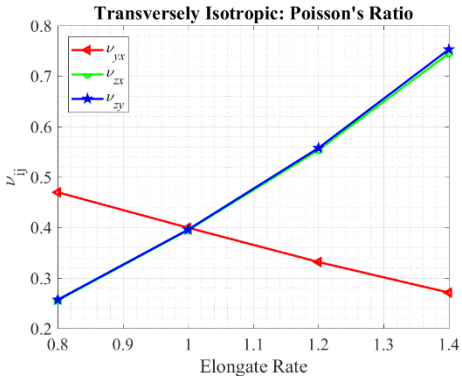
Stiffness

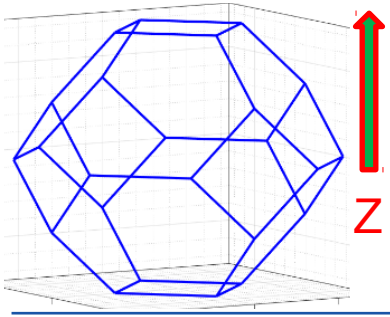


Shear Modulus



Poisson's Ratio

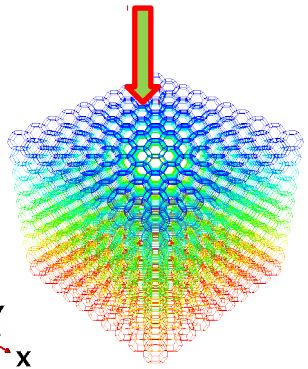


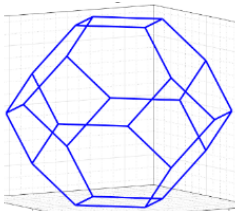


$$H=\begin{bmatrix} H_{11} & H_{12} & H_{13} & & & \\ & H_{22} & H_{23} & & & \\ & & H_{33} & & & \\ & & & H_{44} & & \\ & & & & H_{55} & \\ & & & & & H_{66} \end{bmatrix}$$

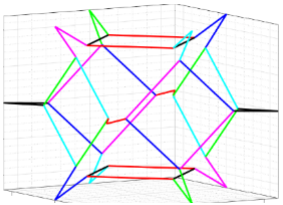
Poisson's Ratio: Positive Vs Negative

Y-Compression



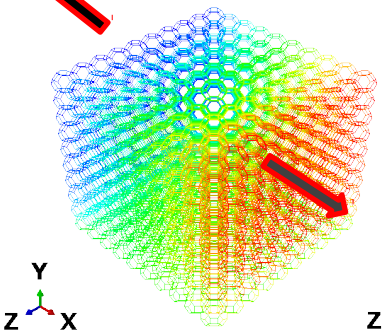


$\nu > 0$

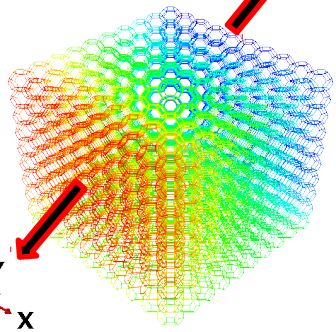


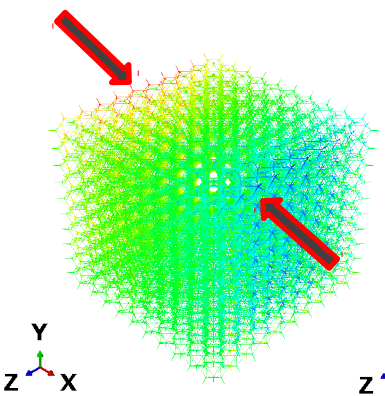
$\nu < 0$

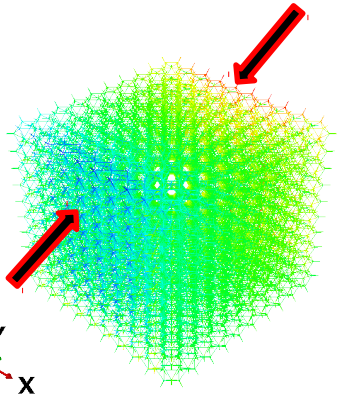
Ux



Uz



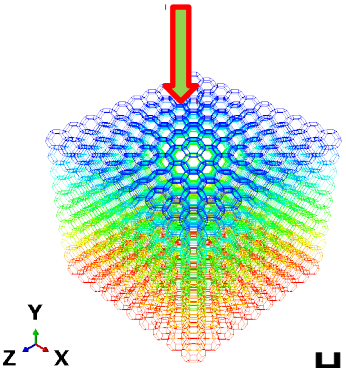


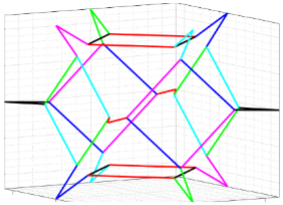


Negative Poisson's Ratio

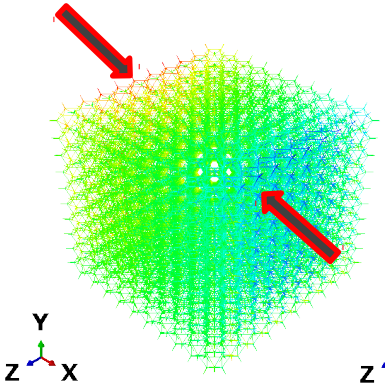
@R. S. LAKES

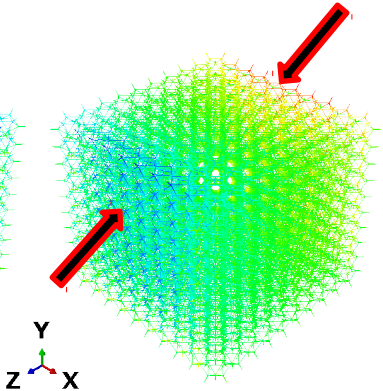
Y-Compression





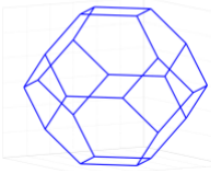
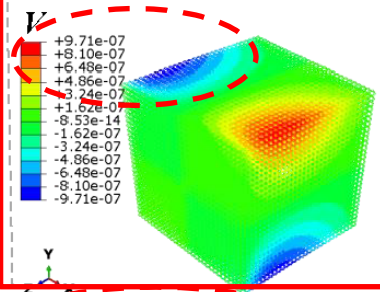
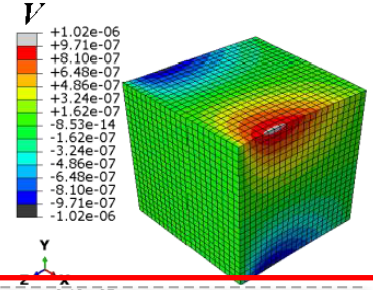
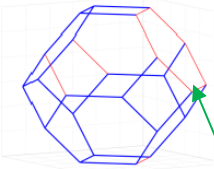
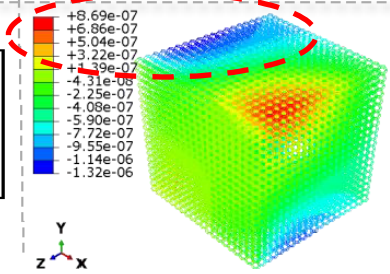
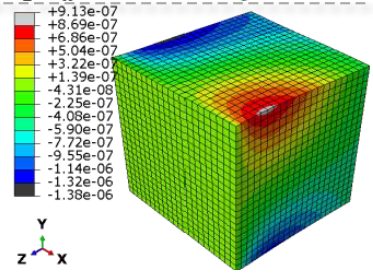
$\nu < 0$





$$H = \begin{bmatrix} 13.8 & -0.27 & -0.26 & 0 & 0 & 0 \\ & 13.7 & -0.26 & 0 & 0 & 0 \\ & & 13.3 & 0 & 0 & 0 \\ & & & 2.4 & 0 & 0 \\ & sym & & & 2.4 & 0 \\ & & & & & 2.4 \end{bmatrix} \text{MPa}$$

Characterize Anisotropic Structures

Single Cell (Kelvin cell)	Hooke's Matrix (MPa) (Estimated static part)	Kelvin cell model (Fixed face Z_{min}; Load u = 0.01 mm on face Z_{max})	Equivalent solid model
 Isotropic	$\begin{bmatrix} 14.6 & 9.6 & 9.6 & 0 & 0 & 0 \\ & 14.6 & 9.6 & 0 & 0 & 0 \\ & & 14.6 & 0 & 0 & 0 \\ & & & 2.6 & 0 & 0 \\ & Sym & & & 2.6 & 0 \\ & & & & & 2.6 \end{bmatrix}$		
 Anisotropic	$\begin{bmatrix} 10.3 & 9.6 & 7.1 & 0.3 & 0.3 & 0.2 \\ & 10.3 & 7.1 & 0.3 & 0.2 & 0.3 \\ & & 10.2 & 0.2 & 0.3 & 0.3 \\ & & & 2.6 & 0.3 & 0.3 \\ & Sym & & & 1.7 & 0.3 \\ & & & & & 1.7 \end{bmatrix}$		

$\neq 0$

Smaller Cross-section Struts

Conclusion

- An inverse method for design and characterization of anisotropic acoustic porous material was developed.
- Microstructure design based on inverse method provides us a possible way to understand and control the material properties during design.
- 3D printed anisotropic viscoelastic foam might be interesting for our future acoustic material design.